FORM TP 2013092

TEST CODE **01234020**

MAY/JUNE 2013

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

22 MAY 2013 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of **TWO** sections.
2. There are EIGHT questions in Section I and THREE questions in Section II.
3. Answer **ALL** questions in Section I, and any **TWO** questions from Section II.
4. Write your answers in the booklet provided.
5. All working must be clearly shown.
6. A list of formulae is provided on page 2 of this booklet.

3092

Required Examination Materials

Electronic calculator
Geometry set
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism: $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder: $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid: $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference: $C = 2\pi r$ where r is the radius of the circle.

Arc length: $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle: $A = \pi r^2$ where r is the radius of the circle.

Area of a sector: $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations: If $ax^2 + bx + c = 0$,

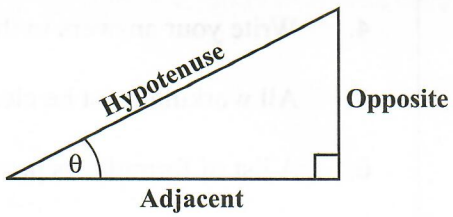
then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios:

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

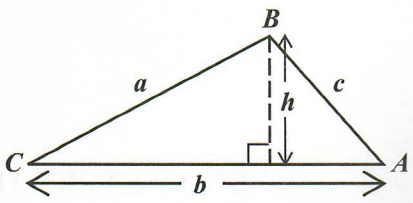


Area of triangle: Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a + b + c}{2}$



Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\dfrac{1\frac{4}{5} - \frac{1}{3}}{2\frac{2}{5}}$ **(2 marks)**

(ii) $\sqrt{1.5625} + (0.32)^2$. **(2 marks)**

(b) Smiley Orange Juice is sold in cartons of two different sizes at the prices shown in the table below.

Carton Size	Cost
350 ml	\$4.20
450 ml	\$5.13

Which size carton of orange juice is the BETTER buy? **Justify your answer.**
(3 marks)

(c) Faye borrowed \$9 600 at 8% per annum compound interest.

(i) Calculate the interest on the loan for the first year. **(1 mark)**

At the end of the first year, she repaid \$4 368.

(ii) How much did she still owe at the beginning of the second year? **(2 marks)**

(iii) Calculate the interest on the remaining balance for the second year. **(1 mark)**

Total 11 marks

2. (a) Factorize completely:

(i) $2x^3 - 8x$ **(2 marks)**

(ii) $3x^2 - 5x - 2$ **(2 marks)**

(b) (i) Make C the subject of the formula $F = \frac{9}{5}C + 32$. **(2 marks)**

(ii) Given that $F = 113$, calculate the value of C. **(1 mark)**

(c) 500 tickets were sold for a concert. Of these x tickets were sold at \$6 each, and the remainder at \$10 each.

(i) Write an expression, in terms of x, for

a) the number of tickets sold at \$10 each **(1 mark)**

b) the TOTAL amount of money collected for the sale of the 500 tickets. **(1 mark)**

(ii) The sum of \$4108 was collected for the sale of the 500 tickets.

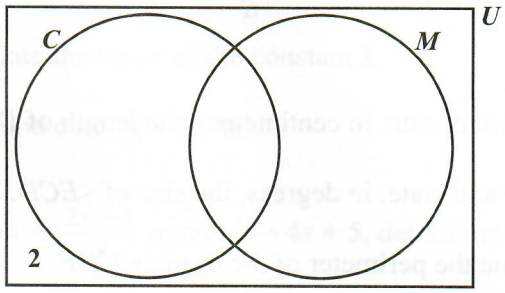
Calculate the number of tickets sold at \$6 each. **(3 marks)**

Total 12 marks

3. (a) A survey of the 30 students in Form 5 showed that some students used cameras (C) or mobile phones (M) to take photographs.

20 students used mobile phones
$4x$ students used ONLY cameras
x students used BOTH mobile phones and cameras
2 students did not use either cameras or phones.

(i) Copy the Venn diagram below and complete it to show, in terms of x, the number of students in each region. **(3 marks)**



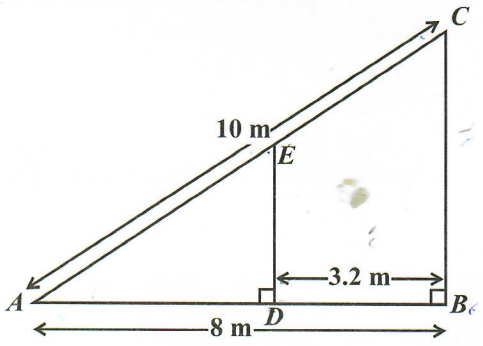
(ii) Write an expression, in terms of x, which represents the TOTAL number of students in the survey. **(1 mark)**

(iii) Determine the number of students in Form 5 who used ONLY cameras. **(2 marks)**

(b) In the diagram below, **not drawn to scale**, AEC and ADB are straight lines.

$<ABC = <ADE = 90°$.

$AC = 10$ m, $AB = 8$ m and $DB = 3.2$ m.



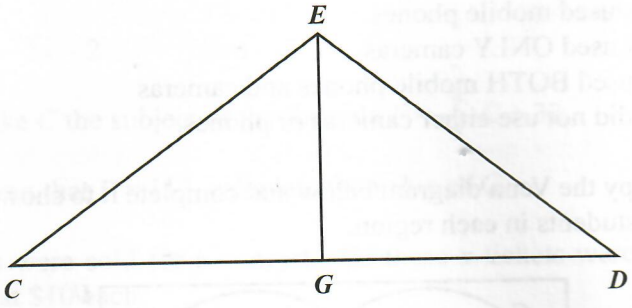
(i) Calculate the length of BC. **(2 marks)**

(ii) Explain why triangles ABC and ADE are similar. **(1 mark)**

(iii) Determine the length of DE. **(3 marks)**

Total 12 marks

4. (a) The diagram below shows an isosceles triangle *CDE*. *G* is the midpoint of *CD*.



(i) Measure and state, in centimetres, the length of *DE*. **(1 mark)**

(ii) Measure and state, in degrees, the size of $\angle ECD$. **(1 mark)**

(iii) Determine the perimeter of the triangle *CDE*. **(2 marks)**

(iv) Calculate the area of the triangle *CDE*. **(1 mark)**

(b) *A* (–1, 4) and *B* (3, 2) are the end points of a line segment *AB*. Determine

(i) the gradient of *AB* **(2 marks)**

(ii) the coordinates of the midpoint of *AB* **(2 marks)**

(iii) the equation of the perpendicular bisector of *AB*. **(3 marks)**

Total 12 marks

5. (a) The incomplete table below shows one pair of values for A and R where A is directly proportional to the square of R.

A	36		196
R	3	5	

(i) Express A in terms of R and a constant k. **(1 mark)**

(ii) Calculate the value of the constant k. **(2 marks)**

(iii) Copy and complete the table. **(2 marks)**

(b) Given that $f(x) = \frac{2x+1}{3}$ and $g(x) = 4x + 5$, determine the values of:

(i) $fg\,(2)$ **(3 marks)**

(ii) $f^{-1}\,(3)$ **(3 marks)**

Total 11 marks

6. (a) A car, travelling along a straight road at a constant speed of 54 km/h, takes 20 seconds to travel the distance between two sign posts.

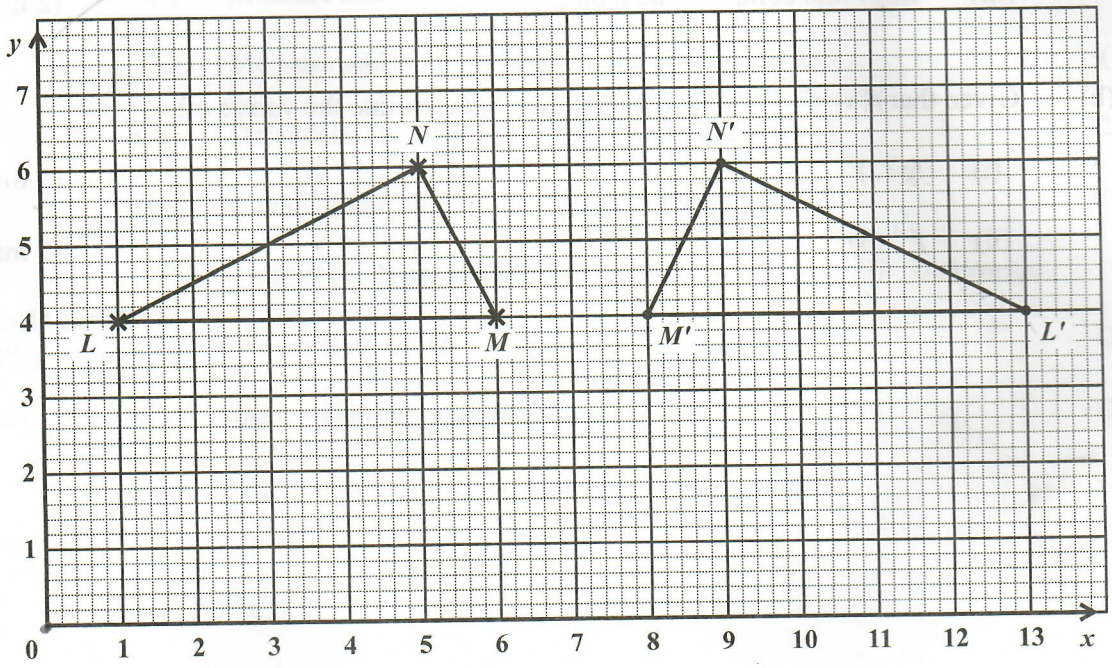
Calculate

(i) the speed of the car in m/s **(2 marks)**

(ii) the distance, in metres, between the two sign posts. **(2 marks)**

(b) **An answer sheet is provided for this question.**

The graph below shows triangle LMN and its image $L'\ M'\ N'$ after undergoing a single transformation.



(i) Describe **fully** the transformation that maps ΔLMN onto $\Delta L'\ M'\ N'$. **(2 marks)**

(ii) **On the answer sheet provided**, draw triangle $L''\ M''\ N''$ the image of triangle LMN, after a translation by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$. **(2 marks)**

(iii) Name and describe a combination of TWO transformations which may be used to map $\Delta L''\ M''\ N''$ onto $\Delta L'\ M'\ N'$. **(3 marks)**

Total 11 marks

7. The table below shows the amount, to the nearest dollar, spent by a group of 40 students at the school canteen during a period of one week.

Amount Spent ($)	Number of Students	Cumulative Frequency
1 – 10	3	3
11 – 20	7	10
21 – 30	9	19
31 – 40	11	30
41 – 50	8	38
51 – 60	2	40

(a) Copy and complete the table to show the cumulative frequency. **(2 marks)**

(b) Using a scale of **1 cm to represent $5 on the horizontal axis** and **1 cm to represent 5 students on the vertical axis**, draw the cumulative frequency graph for the data. **(5 marks)**

(Marks will be awarded for axes appropriately labelled, points correctly plotted, and a smooth curve carefully drawn.)

(c) Use your graph to estimate

(i) the median amount of money spent **(2 marks)**

(ii) the probability that a student chosen at random spent less than $23 during the week. **(2 marks)**

Show on your graph, using broken lines, how these estimates were determined.

Total 11 marks

8. An answer sheet is provided for this question.

The drawings below show the first three diagrams in a sequence.

Diagram 1

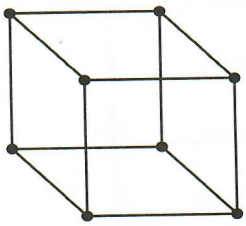


Diagram 2

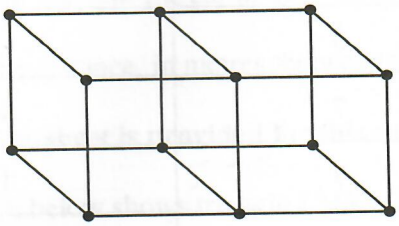
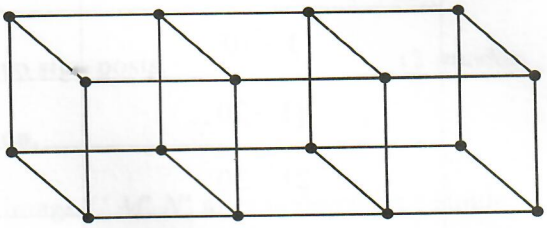
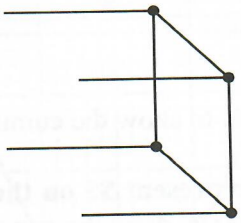


Diagram 3



Each diagram is made up of wires of equal length which are joined at the ends by balls of plasticine. Diagram 1 is made of 12 wires and 8 balls. Each new diagram in the sequence is formed by fitting the frame shown below to the right of the previous diagram.



Thus, Diagram 2 has 8 more wires and 4 more balls than Diagram 1.

On the answer sheet provided:

(a) Draw a sketch of Diagram 4, the fourth diagram in the sequence. **(2 marks)**

(b) Complete the table by inserting the missing values at the rows marked (i) and (ii).

Name of Diagram (N)	**No. of Wires** (W)	**No. of Balls** (B)
1	12	8
2	20	12
3	28	16
(i) 4	_____	_____
(ii) 20	_____	_____

(2 marks)

(4 marks)

(c) Write the rules which may be used to find the values of W and of B where N is known.

(i) W = _____ **(1 mark)**

(ii) B = _____ **(1 mark)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) **An answer sheet is provided for this question.**

Trish wishes to buy x oranges and y mangoes which she intends to carry in her bag. Her bag has space for only 6 fruits.

(i) Write an inequality to represent this information. **(1 mark)**

To get a good bargain, she must buy AT LEAST 2 mangoes.

(ii) Write an inequality to represent this information. **(1 mark)**

More information about the number of oranges and mangoes associated with the good bargain is represented by

$y \leq 2x$.

(iii) Write the information represented by this inequality as a sentence in your own words. **(2 marks)**

(iv) **On the answer sheet provided**, draw the lines associated with the two inequalities obtained in (i) and (ii) above. **(3 marks)**

(v) Shade on your graph the region which represents the solution set for the three inequalities. **(1 mark)**

(b) (i) Write $3x^2 - 12x + 8$ in the form $a(x + h)^2 + k$ where a, h and k are constants. **(3 marks)**

(ii) Sketch the graph of $y = 3x^2 - 12x + 8$, showing on your sketch

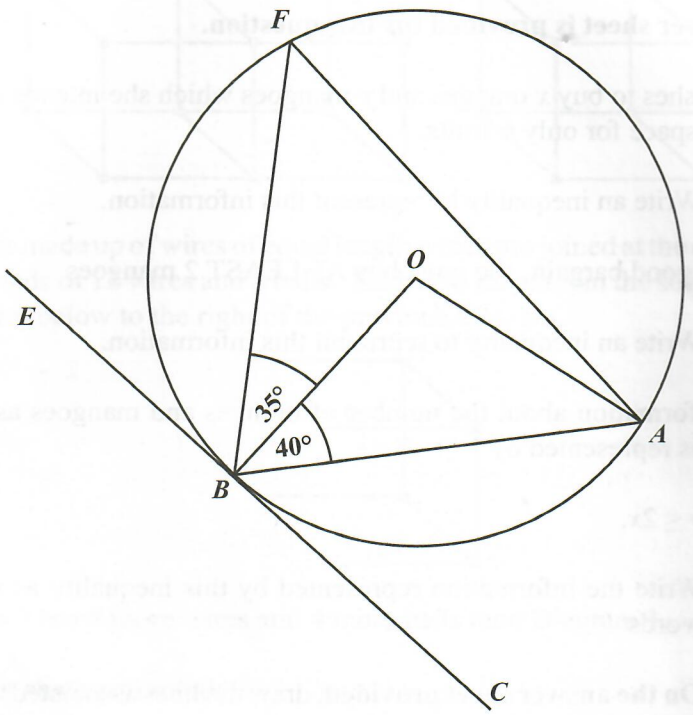
a) the intercept on the y-axis

b) the coordinates of the minimum point. **(4 marks)**

Total 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle with centre O. EBC is a tangent to the circle. $<OBA = 40^\circ$ and $<OBF = 35^\circ$.



Calculate, **giving reasons for your answer**, the measure of

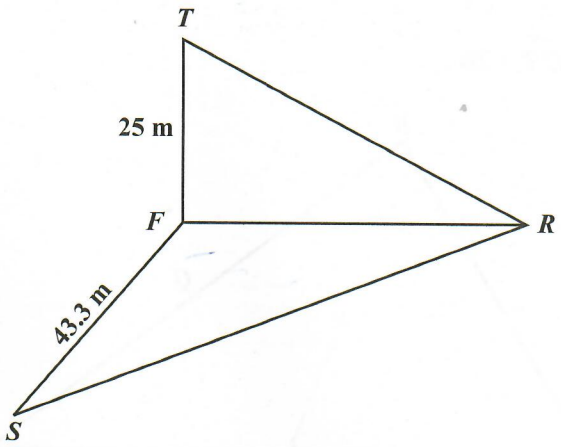
(i) $<EBF$ **(1 mark)**

(ii) $<BOA$ **(2 marks)**

(iii) $<AFB$ **(2 marks)**

(iv) $<OAF$. **(2 marks)**

(b) The diagram below, **not drawn to scale**, shows three points R, S and F on the horizontal ground. FT is a vertical tower of height 25 m. The angle of elevation of the top of the tower, T, from R is 27°. R is due east of F and S is due south of F. $SF = 43.3$ m.



(i) Sketch **separate** diagrams of the triangles RFT, TFS and SFR. Mark on EACH diagram the given measures of sides and angles. **(3 marks)**

(ii) Show, by calculation, that $RF = 49.1$ m. **(2 marks)**

(iii) Calculate the length of SR correct to 1 decimal place. **(1 mark)**

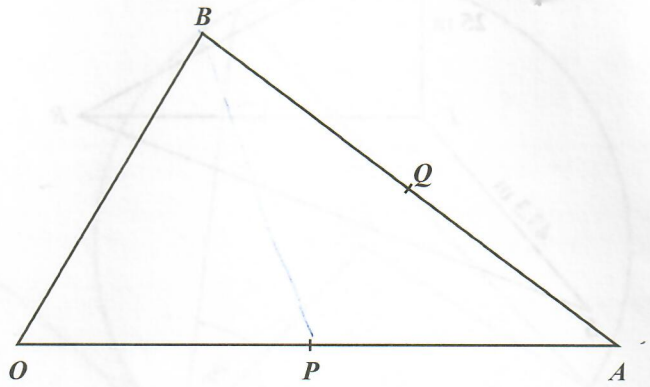
(iv) Calculate the angle of elevation of the top of the tower, T, from S. **(2 marks)**

Total 15 marks

VECTORS AND MATRICES

11. (a) In the diagram below, **not drawn to scale**, P and Q are the midpoints of OA and AB respectively.

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$.



(i) Express in terms of **a** and **b** the vectors

a) $\overrightarrow{AB}$ **(2 marks)**

b) $\overrightarrow{PQ}$. **(2 marks)**

(ii) State TWO geometrical relationships that exist between OB and PQ.
Give reasons for your answers. **(2 marks)**

(b) Given that $M = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

(i) Evaluate M^{-1}, the inverse of M. **(2 marks)**

(ii) Show that $M^{-1} M = I$. **(2 marks)**

(iii) Use a matrix method to solve for r, s, t and u in the equation

$$\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}.$$ **(5 marks)**

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.